Sound field estimation using robotized measurements and the boundary elements method

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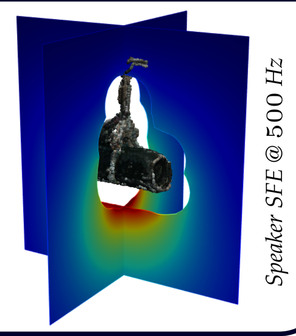
Introduction

Sound Field Estimation (SFE) is a common task in experimental acoustic, which aims to predict the far-field properties of a studied source based on near-field measurements [1]. SFE relies on two aspects: an accurate and versatile method to perform acoustic measurements and an efficient numerical scheme to predict the radiated sound field. Coping with the challenges raised by both aspects, we propose an SFE procedure based on :

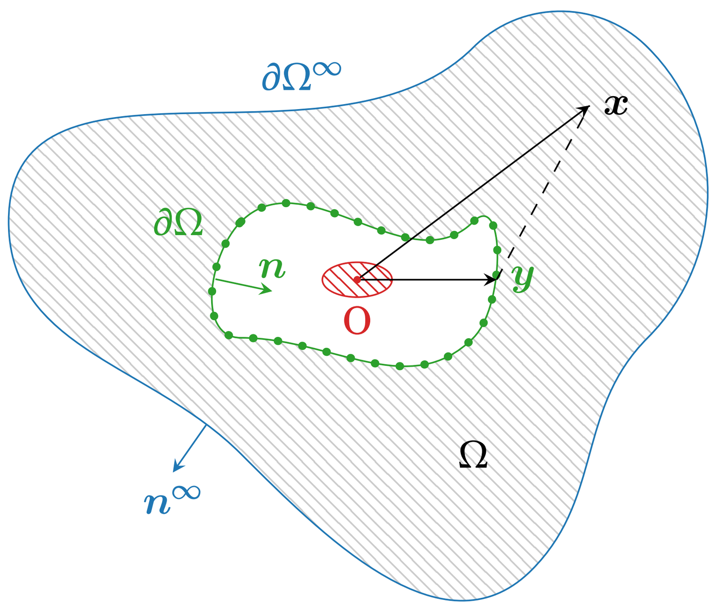
→ A numerical SFE solution derived from the Boundary Elements Method (BEM).

→ Fully- automated 3D acoustic pressure measurements performed with a robotic arm.



The boundary elements method (BEM)

Problem modeling and variational formulation



Sound Field Estimation ⇔ Resolution of the stationary Helmholtz wave equation with the Sommerfeld radiation condition.

If $\partial\Omega$ denotes the closed boundary on which the measurements p_0 are performed, the sought acoustic pressure field p is given by :

$$\begin{cases} \Delta p(\boldsymbol{x}) + k^2 p(\boldsymbol{x}) = 0 & \forall \boldsymbol{x} \in \Omega, \\ p(\boldsymbol{x}) = p_0(\boldsymbol{x}) & \forall \boldsymbol{x} \in \partial\Omega, \\ \lim\limits_{\substack{\boldsymbol{x}\in\partial\Omega^\infty \\ \partial\Omega^\infty \to \infty}} \left(\frac{\partial}{\partial|\boldsymbol{x}|} - ik\right) p(\boldsymbol{x}) = 0, & \end{cases} \quad (1)$$

where $k \in \mathbb{R}$ is the considered wavenumber.

Introducing free-field Green function $G(\boldsymbol{x}, \boldsymbol{y})$, [2] shows that a general solution of (1) is given by the following combined layer potential :

$$\forall\, u : \partial\Omega \to \mathbb{C}, \boldsymbol{x} \in \mathbb{R}^3 \setminus \partial\Omega,\ p(\boldsymbol{x}) = \int_{\partial\Omega} \left(\frac{\partial G(\boldsymbol{x}, \boldsymbol{y})}{\partial \boldsymbol{n}(\boldsymbol{y})} - ikG(\boldsymbol{x}, \boldsymbol{y}) \right) u(\boldsymbol{y}) d\sigma(\boldsymbol{y}) \quad (2)$$

Finding the exact solution comes down to find the boundary density u such that the boundary condition on $\partial\Omega$ is satisfied, *e.g.*, opting for a variational approach :

$$\text{Find } u : \partial\Omega \to \mathbb{C}, \text{ s.t. } \forall\, v : \partial\Omega \to \mathbb{C}, \int_{\partial\Omega} p_0(\boldsymbol{y}')v(\boldsymbol{y}')d\sigma(\boldsymbol{y}') =$$

$$\int_{\partial\Omega} \left[\frac{u(\boldsymbol{y}')}{2} + \int_{\partial\Omega} \left(\frac{\partial G(\boldsymbol{y}', \boldsymbol{y})}{\partial \boldsymbol{n}(\boldsymbol{y})} - ikG(\boldsymbol{y}', \boldsymbol{y}) \right) u(\boldsymbol{y}) d\sigma(\boldsymbol{y}) \right] v(\boldsymbol{y}')d\sigma(\boldsymbol{y}') \quad (3)$$

Numerical resolution and convergence properties

Given a triangular mesh approximating $\partial\Omega$ and choosing P^1 Lagrange elements to describe u and v, (3) may be solved using a Galerkin approximation.

⇒ The measurements p_0 must be performed at each node of the mesh.

Using equation (2), the studied sound field p may then be estimated at any field point of Ω, with an l_2 error decreasing as fast as the squared mesh resolution [3].

Both resolution and prediction steps were implemented using FreeFEM++ BEM library [5]

Robotized measurements

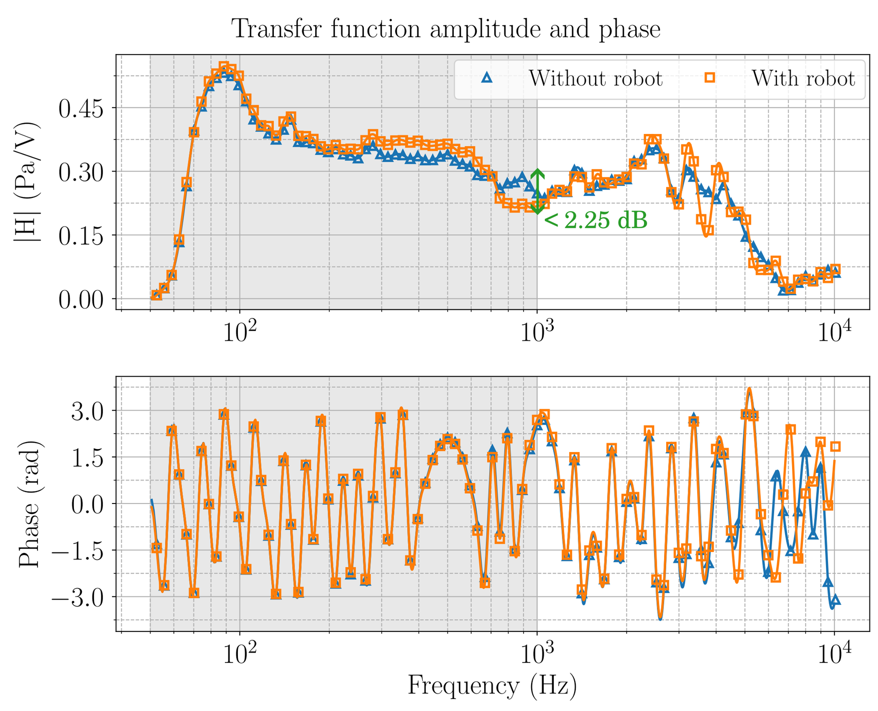
Experimental setup

Validity hypothesis of robotized measurements

→ Sound source stationarity and transfer function computation

As measurements are performed in a sequential manner, our study will focus on the transfer functions of time-invariant electro-acoustic systems ($\Delta < 0.25$ dB).

→ Reflections and scattering caused by the robot



→ How to assess the actual impact of the robot ?

Measurements *with* and *without* the robot, performed in 6 different control configurations.

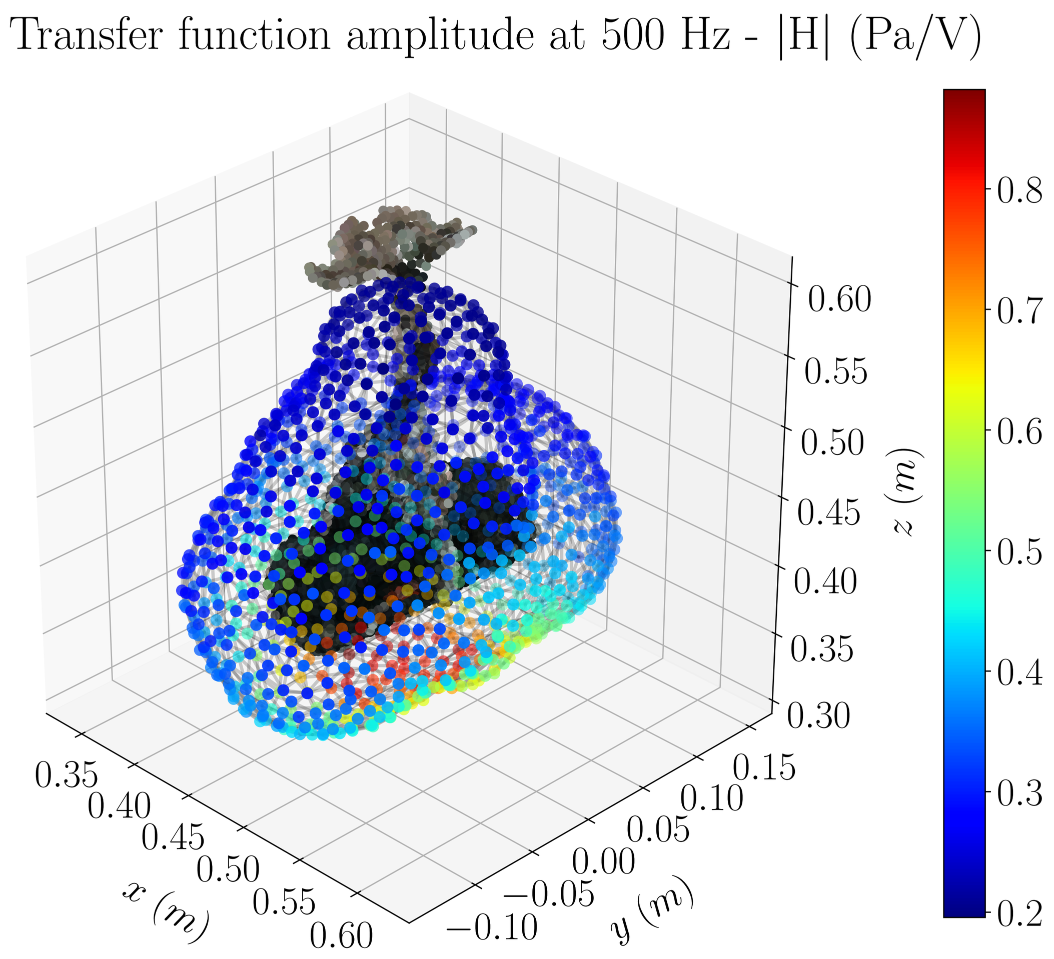
Even in the worst-case scenario, the difference to the robot-less reference remains below 2.25 dB between 50 Hz and 1000 Hz.

15 s white noise signal sampled at 96 kHz, Welch's method and 12^{th} octave smoothing.

→ Flawed positioning accuracy of the robot

Using the calibration procedure presented in [4], the accuracy of the robotic arm was increased to ± 2 mm, ensuring a reliable positioning of the microphone.

Acoustic measurements

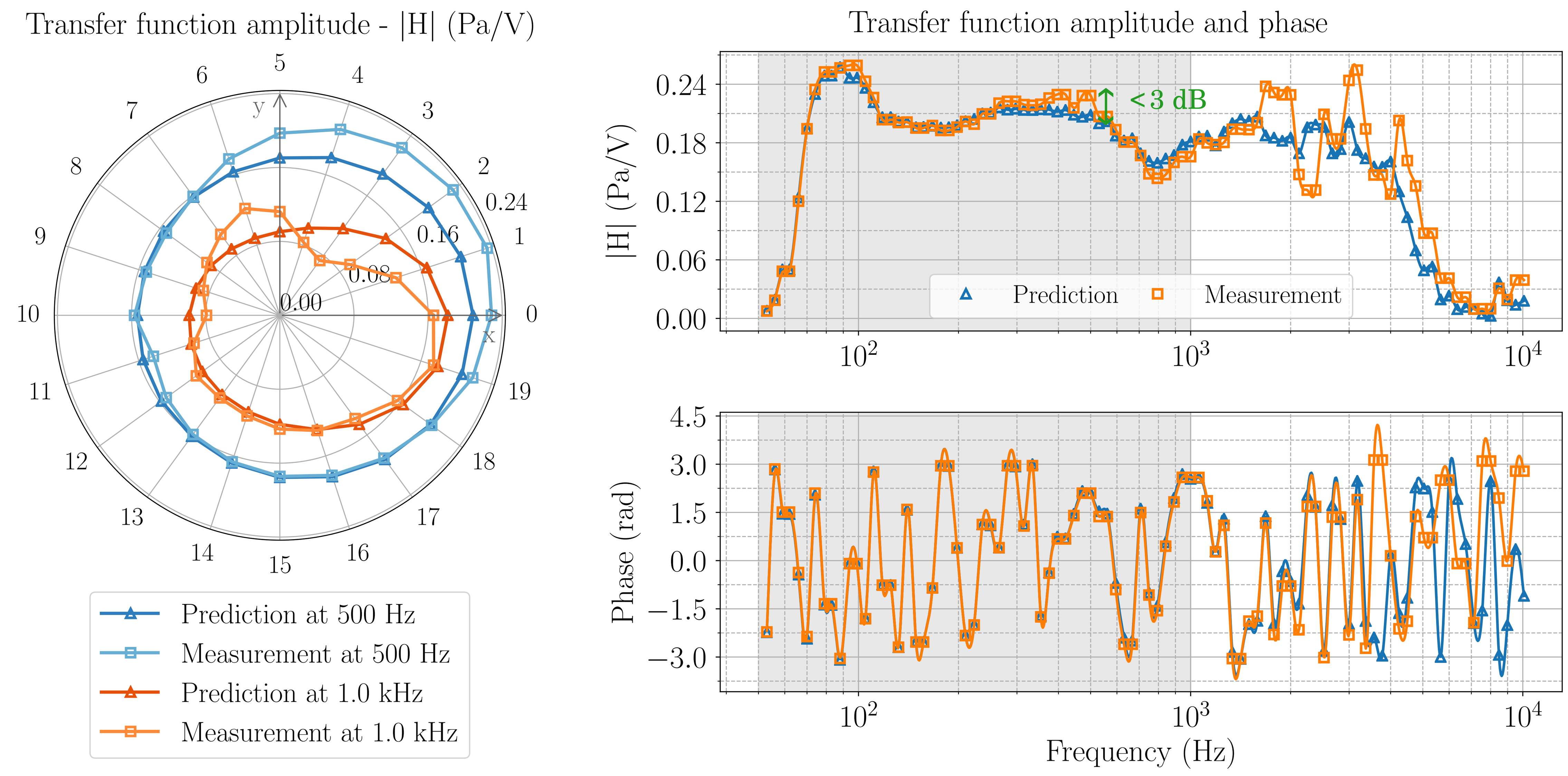


1019 measurements on a conformal surface surrounding a JBL flip 2.

Mesh size $h = 2$ cm - Distance to speaker $d = 5$ cm.

Total acquisition duration $\simeq$ 11 h 30.

Sound Field Estimation results



Estimation results obtained on a circular mesh of radius 25 cm centered on the speaker *(left)* and detailed frequential data for the 0 indexed measurement *(right)*.

References

[1] S. F. Wu, "Methods for reconstructing acoustic quantities based on acoustic pressure measurements", *J. Acoust. Soc. Am.*, 2008, vol. 124, n. 5, p. 2680-2697.

[2] S. A. Sauter et C. Schwab, "Boundary Element Methods", 2011, *Springer Ser. Comput. Math.*, Springer.

[3] J. Giroire, "Integral equation methods for the Helmholtz equation", *Integral Equations and Operator Theory*, 1982, vol. 5, n. 1, p. 506-517.

[4] C. Pascal, O. Doaré and A. Chapoutot, "A ROS-Based Kinematic Calibration Tool for Serial Robots", *2023 IEEE Int. Conf. Intell. Robots Syst. (IROS)*, 2023, Detroit, MI, USA, p. 1767-1773.

[5] F. Hecht, "New development in FreeFem++", *J. Numer. Math.*, 2012, vol. 20, n. 3-4, p. 251-266.

Perspectives

→ Further improve BEM convergence properties with higher-order geometric elements;

→ Investigate practical and numerical methods for the reduction of the robot acoustic footprint;

→ Extend the procedure to the resolution of inverse Nearfield Acoustic Holography (NAH) problems.

Check out our code !